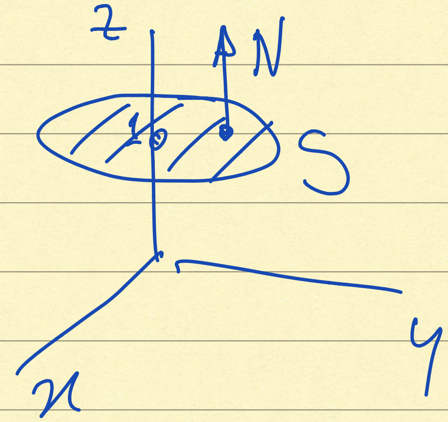
# Rotacional. Teorema de Stokes

$F:\mathbb{R}^3 \to \mathbb{R}^3$, $C^1$ , $F=(P,Q,R)$

$$\operatorname{rot} F = \left(\frac{\partial R}{\partial y}-\frac{\partial Q}{\partial z},\ \frac{\partial P}{\partial z}-\frac{\partial R}{\partial x},\ \underbrace{\frac{\partial Q}{\partial x}-\frac{\partial P}{\partial y}}\right)$$

$$= \text{"}\begin{array}{l} \\ \nabla \to \\ F \to \end{array}\begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ P & Q & R \end{vmatrix}\text{"} \equiv \nabla \times F$$

$$S = \{(x,y,z)\in\mathbb{R}^3 : \boxed{z=1},\ x^2+y^2<1\}$$

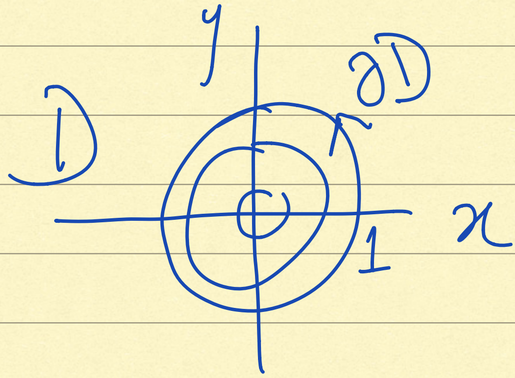



$$\iint_S \underbrace{\operatorname{rot} F \cdot N}$$

$N=(0,0,1)$

$$\operatorname{rot} F \cdot N = \underbrace{\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}} \quad (\text{Green})$$

$$S: \; g(x,y) = (x, y, 1) \; ; \; \underbrace{x^2+y^2<1}_{D}$$




$$D_x g = (1,0,0) = i$$

$$D_y g = (0,1,0) = j$$

$$D_x g \times D_y g = (0,0,1) = k$$

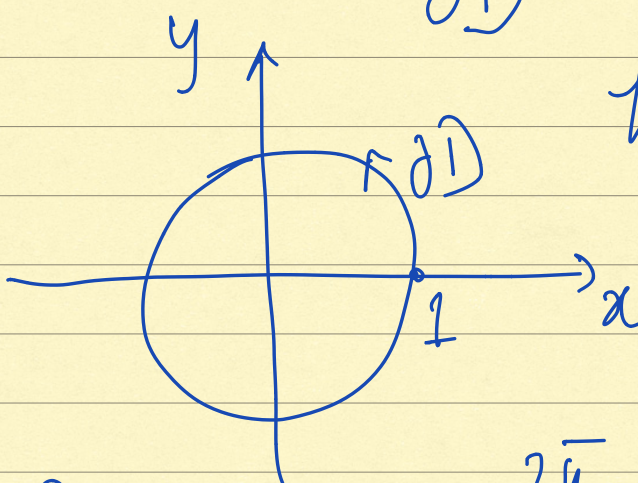
$$\iint_S \operatorname{rot} F \cdot N = \iint_D \operatorname{rot} F(g(x,y)) \cdot D_x g \times D_y g \, dx\,dy$$

$$= \iint_D \left( \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dx\,dy =$$

pelo T. de Green,

$$\iint_S \operatorname{rot} F \cdot N = \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dx\,dy$$

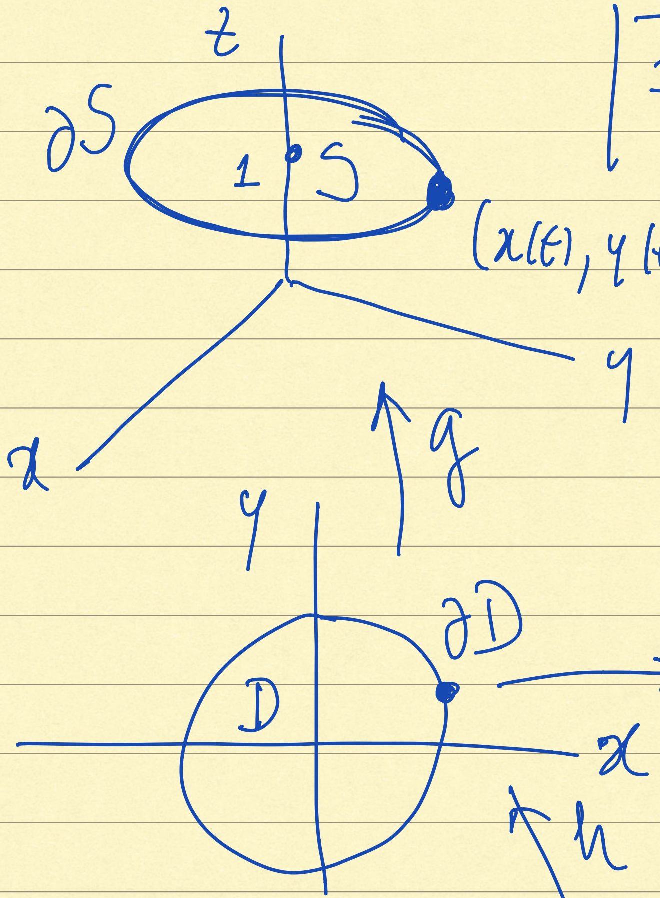
$$= \oint_{\partial D} P\,dx + Q\,dy$$




$$h(t) = (\cos t, \operatorname{sen} t, 0) = (x(t), y(t), 0)$$

$$0 \le t \le 2\pi$$

$$\oint_{\partial D} P\,dx + Q\,dy = \int_0^{2\pi} \Big(P(g(t))\,x'(t) + Q(g(t))\,y'(t)\Big)\,dt$$

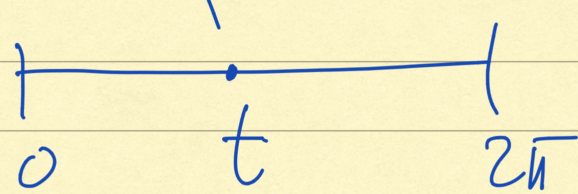



$$\boxed{z = 1,\ x^2 + y^2 = 1}$$

$$(x(t), y(t), 1) = g(h(t))$$

$$\partial S = g(\partial D)$$

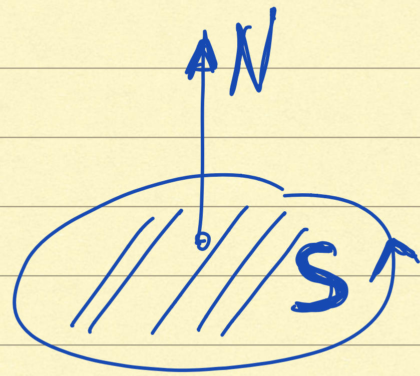
$$h(t) = (x(t), y(t), 0)$$




$$\int_{\partial D} P\,dx + Q\,dy = \int_0^{2\pi} P(x(t), y(t), 0)\, x'(t) + Q(x(t), y(t), 0)\, y'(t)$$

$$= \int_{\partial S} P\,dx + Q\,dy + \underbrace{R\,dz}_{=0}$$

$$\therefore \iint_S \operatorname{rot} F \cdot N = \int_{\partial S} F \cdot d\gamma$$




$\partial S = \boxed{\text{Bordo de } S}$

Fluxo do rotacional de $F$ em $S$ = Trabalho de $F$ em $\partial S$

## Bordo de uma superfície em $\mathbb{R}^3$

1 equação + inequação

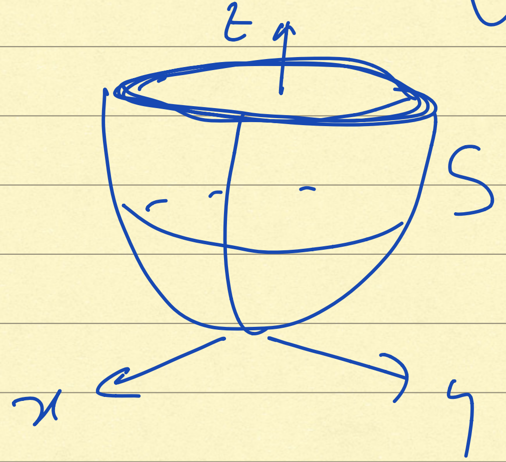
$\Downarrow$

equações.

Exemplo: 1) $S: \quad z = 1, x^2 + y^2 \leq 1$

$\partial S: \quad z = 1 ; x^2 + y^2 = 1$

2) $S: \quad z = x^2 + y^2 < 1$.

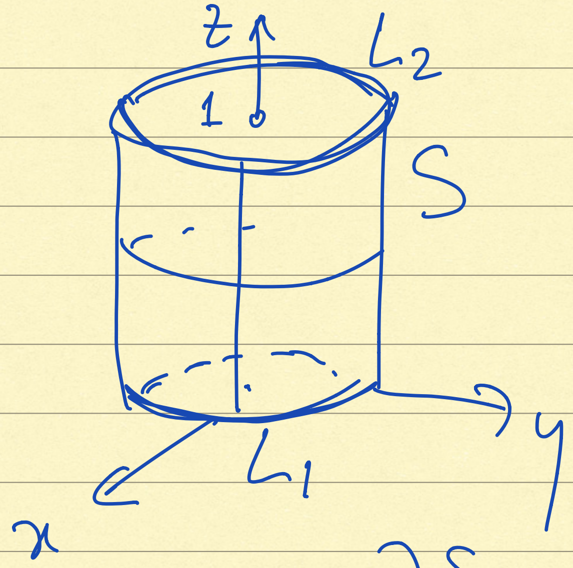



$D \quad =$

$\partial S: \quad z = x^2 + y^2 = 1$

$$\begin{cases} z = x^2 + y^2 \\ z = 1 \end{cases}$$

3) $S: \quad x^2 + y^2 = 1 ; \quad 0 < z < 1$



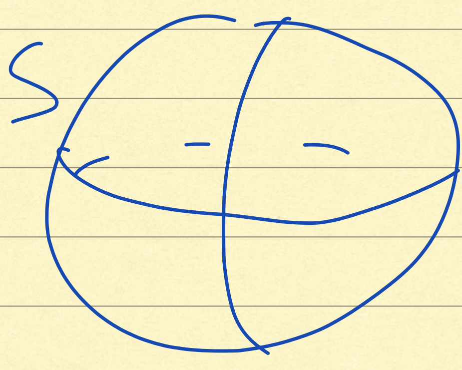

$x^2+y^2=1;\ \underline{0<z<1}$

$\partial S : \{x^2+y^2=1;\ z=0\}$

$\cup \{x^2+y^2=1,\ z=1\}$

$\partial S = L_1 \cup L_2.$

4) $S: x^2+y^2+z^2=1$




$\partial S = \phi$

Aplicação: Ficha 13

1- Dados: $F(x,y,z) = ( \cdots )$

$S: 0 \leq z = x^2+y^2-1 \leq 3$

$N$, $N_3 < 0$

$$\iint_S \text{rot} F \cdot N = \iint_S \nabla \times F \cdot N \quad ??$$

$$\text{Stokes} = \underbrace{\oint_{\partial S} F \cdot d\gamma}_{\text{Calcular}}$$

$\therefore$ identificar $\partial S$, parametrizar e calcular.